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Modeling the Behavior of a Quantum Well Utilizing the P-N Junction
Research Report

Introduction

The study of quantum mechanical structures is a fascinating branch of science in which there is still much to be discovered and many innovations to be developed. Seeing the potential for research in this field, this line of study was chosen as a means to provide a basis to eventually continue work in this discipline through the collegiate level and onwards. Specifically, the topic of quantum computing, the idea that, in theory, each individual electron could function as a transistor through the utilization of spin states, was attractive due to the potential for exponentiating the processing power of a computer (DiVincenzo, 2006). Whereas modern day transistors run on the premise of 1's and 0's to convery information, quantum computing purports that electron transistors would run based on the "up" and "down" spins of an electron (DiVincenzo, 2006 & Hameka, 1981). Some scientists even theorize it will eventually be possible to increase computational capacity even more through a superposition of these two states in the form of a quantum bit, or qubit (Choi, 2005).

Present research shows that controlling the flip of an electron may be achieved using induced methods. In 2001, scientists at the Center for Spintonics and Quantum Computation at the California NanoSystems Institute (CNSI) claim that the process of "induced precession" can over rotate the electron and thereby alter its spin. CNSI developed a spin gate which can function "across a continuum" rather than turning the electrons "on and off" (Savani, 2001). Four years later, physicists at Ohio University were able to induce an electron to change its direction by exposing an electron in a quantum dot to a small voltage (*Physicists*, 2005). Both of these studies indicate that the most probable avenue to future electron control lies in the manipulation of magnetic fields. What this progression of research over time also shows is that the key to capturing these electrons to be examined lies in quantum nanostructures. CNSI made a point to note that quantum wells would be essential in further research towards fully controlling electrons (Savani, 2001). Quantum wells (two and three-dimensional) are one of the simplest types of quantum structures being currently studied along with quantum strings (two-dimensional) and quantum dots (one-dimensional). Traditionally, quantum wells are created at the interface of semiconductors which are arranged in such a way that electrons resulting from doped material are trapped in a potential "well." (Frensley, 1995 & Speziale, 2005) Currently, some popular types of semiconductors conducive to this end are gallium arsenide (GaAs) and aluminum gallium arsenide (AlGaAs), both used by CNSI, and indium arsenide (InAs), used by Ohio University, due to their varying band gaps, the distance between the valence and conduction band of electrons (Kayali, 2002). The variation in band gap length causes electrons to diffuse across the varying band gap interface in one direction. By sandwiching a small band gap semiconductor with large band gap semiconductors, electrons are induced into the material of smaller band gap and essentially trapped (Schubert, 2003). The task then lies in knowing the characteristics of those electrons in the quantum well. Due to the Heisenberg uncertainty principle, one is unable to know both the position and momentum of an object, in this case an electron, at the same time (Hilgevoord & Uffink, 2001). Therefore, to define the next best possible measurement would involve a study of the probability of determining an object being in a given place at a given time.

Not surprisingly, at the start of this study, minimal research was found regarding the constructing and analysis of actual quantum wells. While this made establishing a baseline knowledge of quantum wells all the more challenging, this fact was one more piece of evidence that this was a field of research that would be still applicable to modern innovation many years into the future and therefore a field open for making novel strides. One finding of note, however, came from Sandia National Laboratories where a quantum mechanical transistor had been created using the tunneling techniques of quantum mechanics (Simmons, 1998). Aside from this, the most persistent information found regarded the “particle in a box” concept. That is, the solution of the Schrödinger equation for a three sided structure (Furth, 1970). Schrödinger’s equation is one of the fundamental mathematical principles governing the modern interpretation of non-relativistic quantum mechanics. What Newton’s second law is to classical mechanics, Schrödinger’s equation is to quantum mechanics by describing the relationship between space and time in those systems (Fong, 1962). The revolutionary aspect of this singular equation is the incorporation of the particle-wave duality principle, the concept that all objects in the universe exhibit properties of both particles, discrete energy levels, and waves, interference. Schrödinger’s equation is uniquely applicable to both situations though only one will be addressed in this experiment (Liboff, 1908). Following “the particle in a box” tangent lead logically to applying the Schrödinger equation to a box shaped quantum well (Van Zeghbroeck, 2004).

Purpose

The purpose of this research is to study quantum nanostructures in order to find a logical method in which to experiment with the characteristics of these devices and thereby establish them as viable components of future innovations. Two studies were conducted to achieve this end over two years. First, the experiment sought to determine comparative techniques for time-invariant quantum well analysis using Schrödinger’s equation within the simplest type of quantum structure, the one-dimentional infinite potential quantum well. The one-dimensional infinite structure was chosen due to its mathematical simplicity and since it would be the first logical stepping stone towards acquiring a full understanding of quantum phenomena by working from the ground up. The first study also aimed to incorporate Born's Probability Density Function (Acedo, n.d.). Thus, the project would evaluate the probability that an electron was in a given region of an infinite potential quantum well at different discrete energy levels. Secondly, the experiment to wanted test if the properties of a general purpose diode could parallel an aspect of the infinite potential quantum well and thereby prove its feasibility in physically modeling some aspect of the general solution to the Schrödinger equation.

With these goals in mind, it was hypothesized that a technique for solving the time-invariant Schrödinger's Equation could be achieved using model methods written using the Matlab/Simulink program (Matlab, 2005). It was also hypothesized that the model technique would be able to incorporate the Probability Density Function for a given region of an infinite potential quantum well at different discrete energy levels. The results of which would be analogous to empirical data found through analytical calculations.

The second aspect of the research, aimed to find a physical structure with characteristics mimicing those of the quantum well. The logical first step was to research semiconductor devices to find out what kinds of electrical components are based on the interaction of semiconductor materials (Gavryushin & Zukauskas, 2002). It was quickly found that general purpose diodes are made from two semiconductors whose function is based on the difference in band gap at the interface of the two materials at the p-n junction (Birner, 2007). This being the same type of system at work in quantum well formation, it was only logical that the general purpose diode would be the optimal device to attempt to replicate quantum well characteristics. It was hypothesized that utilizing the p-n junctions found in general purpose diodes, a model that mimics the change in probability density of finding an electron in different discrete energy levels of an infinite potential quantum well, could be achieved. This would be proven experimentally through modelling the current versus voltage output of diodes tested against varying resistor values and analyzing the nonlinear aspects of the system (*Diode Tutorial*, n.d.). It was thought that the nonlinear facets would correlate to the nonlinear changes in probability density at varying discrete energy levels.

Experimentation

Beginning with the first part of the study, derivative and integral calculus were studied through a mentor in order to acquire the necessary skills to solve the Schrödinger Equation. The analytical procedure for solving the time-invariant Schrödinger Equation for a one dimensional quantum well was as follows:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\boldsymbol{\psi}(x,y,z,t)}{\partial x^2}+U(x,y,z)\boldsymbol{\psi}(x,y,z,t)=i\hbar\frac{\partial\boldsymbol{\psi}(x,y,z,t)}{dt} \quad \textit{Equ. 1}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\boldsymbol{\psi}(x,t)}{\partial x^2}+U(x)\boldsymbol{\psi}(x,t)=i\hbar\frac{\partial\boldsymbol{\psi}(x,t)}{\partial t} \quad \textit{Equ. 2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\boldsymbol{\psi}(x,t)}{\partial x^2}=i\hbar\frac{\partial\boldsymbol{\psi}(x,t)}{\partial t} \quad \textit{Equ. 3}$$

$$\boldsymbol{\psi}(x,t)=\Psi(x)\Phi(t) \quad \textit{Equ. 4}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}\Phi(t)=i\hbar\Psi(x)\frac{d\Phi(t)}{dt} \quad \textit{Equ. 5}$$

$$\frac{-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}\Phi(t)}{\Psi(x)\Phi(t)}=\frac{i\hbar\Psi(x)\frac{d\Phi(t)}{dt}}{\Psi(x)\Phi(t)} \quad \textit{Equ. 6}$$

$$\frac{-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}}{\Psi(x)}=E=\frac{i\hbar\frac{d\Phi(t)}{dt}}{\Phi(t)} \quad \textit{Equ. 7}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}=E\Psi(x) \quad \textit{Equ. 8}$$

$$\frac{\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}+E\Psi(x)=0 \quad \textit{Equ. 9}$$

$$\frac{d^2\Psi(x)}{dx^2}+\frac{2mE}{\hbar^2}\Psi(x)=0 \quad \textit{Equ. 10}$$

$$\Psi=Be^{-i\sqrt{\frac{2m}{\hbar^2}E}x} \quad \textit{Equ. 11}$$

$$\frac{d\Psi}{dx}=(-i\sqrt{\frac{2mE}{\hbar^2}})Be^{-i\sqrt{\frac{2mE}{\hbar^2}}x} \quad \textit{Equ. 12}$$

$$\frac{d^2\Psi}{dx^2}=(-\frac{2mE}{\hbar^2})Be^{-i\sqrt{\frac{2mE}{\hbar^2}}x} \quad \textit{Equ. 13}$$

$$(-\frac{2mE}{\hbar^2})Be^{-i\sqrt{\frac{2mE}{\hbar^2}}x}+(\frac{2mE}{\hbar^2})Be^{-i\sqrt{\frac{2mE}{\hbar^2}}x}=0 \quad \textit{Equ. 14}$$

$$\Psi(x) = Be^{-i\sqrt{\frac{2m}{\hbar^2}Ex}} + Ce^{i\sqrt{\frac{2m}{\hbar^2}Ex}} \quad \textit{Equ. 15}$$

$$\Psi(x) = Be^{-ikx} + Ce^{ikx} \quad \textit{Equ. 16}$$

$$\Psi(0) = 0 = B + C \Rightarrow B = -C \quad \textit{Equ. 17}$$

$$\Psi(x) = -Ce^{-ikx} + Ce^{ikx} \quad \textit{Equ. 18}$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \quad \textit{Equ. 19}$$

$$\Psi(x) = (2iC)\sin(kx) \quad \textit{Equ. 20}$$

$$\Psi(L) = (2iC)\sin(kL) = 0 \quad \textit{Equ. 21}$$

$$\sin(kL) = 0, \text{ then } kL = n\pi,\ n = 1,2,3\ldots \quad \text{so, } k = \frac{\text{n}\pi}{\text{L}} \quad \textit{Equ. 22}$$

$$\Psi(x,t) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})e^{-i\frac{E}{\hbar}t} \quad \textit{Equ. 23}$$

$$P = \Psi(x,t)\Psi^*(x,t) \quad \textit{Equ. 24}$$

$$\Psi^*(x,t) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})e^{i\frac{E}{\hbar}t} \quad \textit{Equ. 25}$$

$$\Psi \cdot \Psi^* = \frac{2}{L}\sin^2(\frac{n\pi x}{L}) \quad \textit{Equ. 26}$$

$$p(a < x < b) = \int_a^b \frac{2}{L}\sin^2(\frac{n\pi x}{L})dx \quad \textit{Equ. 27}$$

$$p(a < x < b) = \frac{2}{L}\int_a^b [\frac{1 - \cos(\frac{2n\pi x}{L})}{2}]dx \quad \textit{Equ. 28}$$

$$p(a < x < b) = \frac{2}{L}\int_a^b [\frac{1}{2} - \frac{1}{2}\cos(\frac{2\pi nx}{L})]dx \quad \textit{Equ. 29}$$

$$= \frac{2}{L}\int_a^b \frac{1}{2}dx - \frac{2}{L}\int_a^b \frac{1}{2}\cos(\frac{2\pi nx}{L})dx \quad \textit{Equ. 30}$$

$$= \frac{1}{L}\int_a^b dx - \frac{1}{L}\int_a^b \cos(\frac{2\pi nx}{L})dx \quad \textit{Equ. 31}$$

$$= \frac{1}{L}[x]_a^b - \frac{1}{L}\frac{L}{2\pi n}[\sin(\frac{2n\pi x}{L})]_a^b \quad \textit{Equ. 32}$$

$$= \frac{1}{L}[x]_a^b - \frac{1}{L}\frac{L}{2\pi n}[\sin(\frac{2n\pi x}{L})]_a^b \quad \textit{Equ. 33}$$

$$= \frac{1}{L}[b - a] - \frac{1}{2\pi n}[\sin(\frac{2n\pi b}{L}) - \sin(\frac{2n\pi a}{L})] \quad \textit{Equ. 34}$$

$$= [\frac{b}{L} - \frac{a}{L}] - \frac{1}{2\pi n}[\sin(\frac{2n\pi b}{L}) - \sin(\frac{2n\pi a}{L}) \quad \textit{Equ. 35}$$

$$p(a < x < b) = [\frac{b - a}{L}] - \frac{1}{2\pi n}[\sin(\frac{2n\pi b}{L}) - \sin(\frac{2n\pi a}{L})] \quad \textit{Equ. 36}$$

Equations 1-36:
General solution to the Schrödinger equation for a one-dimensional infinite potential quantum well.

The result was a general equation which could be used to calculate the probability of an electron residing in a specific region of an infinite potential quantum well if all of the following were known: the discrete energy level to be studied, n, the length of the quantum well, L, and the upper, a, and lower, b, limits for the region of the well to be analyzed. Each trial from this part of the procedure for the first study was defined by studying a specific region of the theoretical well for a defined region in a discrete energy level. This was conducted for each of the first ten discrete energy levels with the arbitrary decision to analyze the middle fifty-percent of the quantum wells (quality control trials ensured the equations were effective for all regions, the center was simply chosen as the region of interest to analyze).

Using the information gleaned in the process of solving Schrödinger's equation, a program was created utilizing the Simulink component of Matlab (Fig. 1a) (Matlab, 2005). The format consisted of a three subprogram structure in which the three individual programs would essentially define the three major analytical steps in solving the time-invariant Schrödinger equation. The three subprograms, respectively, were written to: define the general solution to the time-invariant Schrödinger equation (Fig. 1b), define the limits of the infinite potential quantum well where the probability of finding an electron would be calculated (Fig. 1c), and implement the probability density function to calculate the probability that an electron would be in a given region of the quantum well (Fig. 1d). As a mirror to the analytical procedure, the program was designed such that the variables needed to solve the general solution to the Schrödinger equation were the same variables set as parameters into the program to obtain the same result (n, L, a, b). The final programs appeared as follows:

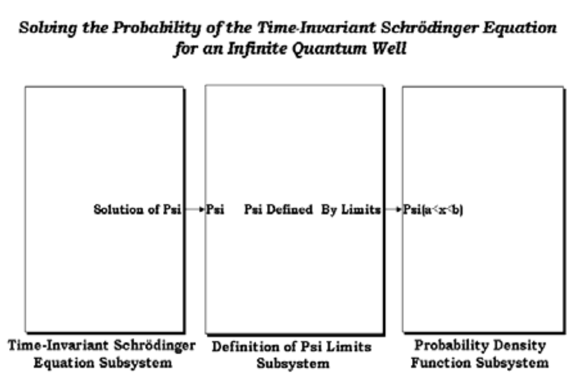

Figure 1a: Program to solve Schrödinger's equation,

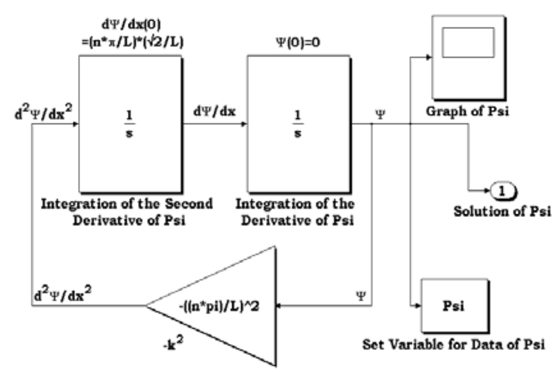

Figure 1b: Solve for a general solution to Schrödinger's equation.

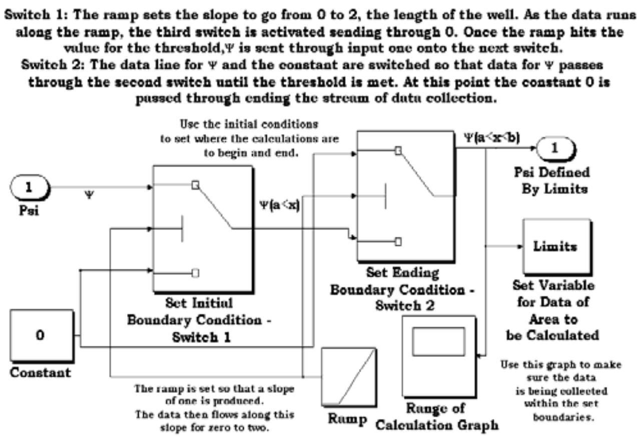

Figure 1c: *Define the limits of the quantum well in which to calculate the probability density.*

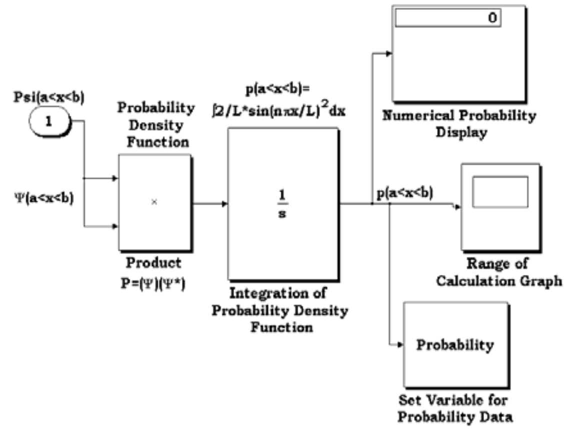

Figure 1d: Implement the probability density function.

Figure 1:
Simulink model program to solve the time-invariant Schrödinger equation for a one dimensional infinite potential quantum well.
Defined by three subprograms (b), (c), and (d).

The result of the first study was a sucessful determination of both the analytical and model techniques for solving the time-invariant Schrödinger equation. The two methods were found to produce equivalent results for the probability of an electron in a given region of an infinite potential quantum well. The final outputs from this program can be seen in the section of this report discussing the analysis of these graphs.

Armed with a program able to output the probability density of an electron being within different regions of an infinite potential quantum well, the characteristics of the probability density functions were studied in order to find a device whose characteristics could mimic the probability densities. Knowing that p-n junctions exhibit non-ohmic characteristics in the form of exponential regions in the current versus voltage curve, when defined as a piecewise function, these structures were examined to find a similarity between the exponential regions of the probability densities and those of the p-n junction diodes (*Diode Tutorial*, n.d.).

To begin with, the use of a high school physics laboratory was negotiated with the consent of the school's physics instructor who provided many of the staple materials used throughout this part of the experiment. From the laboratory, breadboards, jump wires, resistors, cables, AC/DC power sources, and digital multi-meters as well as Vernier Software Labpro current and voltage probes and the accompanying software were easily obtained. Ten different general purpose silicon diodes were obtained from a local electrical supply store. In keeping with uniformity, the supplier was consulted to ensure that the diodes were made from the same material, in this case silicon. Most of the diodes used came from the 1N diode series. After experimentation had begun, it was determined that oscilloscopes (to validate the efficiency of the circuit) and stronger power sources would be needed. These materials were acquired on loan from a local community college.

The setup used to collect data involved a simple circuit driven by a direct current running into a breadboard consisting of a diode (the direction dependent upon the type of trial being conducted) and a resistor (or varying resistance) all of which were in series to the power supply. An oscilloscope was used to read the circuit and visually ensure all components were functioning appropriately. The oscilloscope was verified to be calibrated before use to ensure accurate readings. Each trial was defined as taking one of the diodes from one of the ten types (using the same diode for the duration of the trial) and placing it into the circuit in the forward-biased direction. Digital multi-meters were set up at intervals along the circuit to read for current and voltage. The power supply was engaged and a current passed through the circuit and current and voltage readings were recorded along with the value of the resistor. This procedure was repeated for sixty-three different resistor values, again, all used on the same diode. The resistor values used were: 10Ω, 100Ω to 1kΩ increasing by 50Ω, 1kΩ to 10kΩ increasing by 500Ω, 10kΩ to 50kΩ increasing by 5kΩ, 100kΩ to 1MΩ increasing by 500kΩ, 5MΩ and 10MΩ. After the forward-biased part of the trial was completed, the diode was changed to the reverse-biased direction and the same procedure conducted again using the same sixty-three values for the resistors. Sixty-three was simply the most combinations that could be created with the resistors available and allowed a wide range of data to be collected without leaving large gaps once the data was graphed. The sixty-three forward and reverse-biased values were recorded for each of the ten different types of diodes. The data from the forward and reverse-biased collections were then graphed together for each of the ten diodes to obtain two current versus voltage graphs. These graphs were then analyzed and compared to the graphical output of the Simulink programs for the first ten discrete energy levels. The data collected from all ten diodes and their resultant graphs can be seen in the section concerning analysis of this data.

Analysis

At this point, the fact that both the probability densities of the electron's positioning in a one-dimensional infinite potential quantum well and the current versus voltage diode outputs had exponentially defined regions had been experimentally established. To strengthen the results of this experiment, it was decided to mathematically correlate the results by deriving a general solution from the exponential regression equations. The output from the first study was reexamined using exponential regression curve fits for the exponential regions of all ten discrete energy levels. The LoggerPro program used to graph the results had a built in regression curve fit function. However, at times, a manual fit actually returned a more accurate exponential curve fit. The graphs (Figs. 2a-2j) which follow are based on the parameters that the wells are of length 2, chosen for mathematical ease, and the probability density is being calculated for the center fifty-percent of the wells. The graphs represent the probability of finding the electron (dependent variable) versus the position in the quantum well (independent variable).

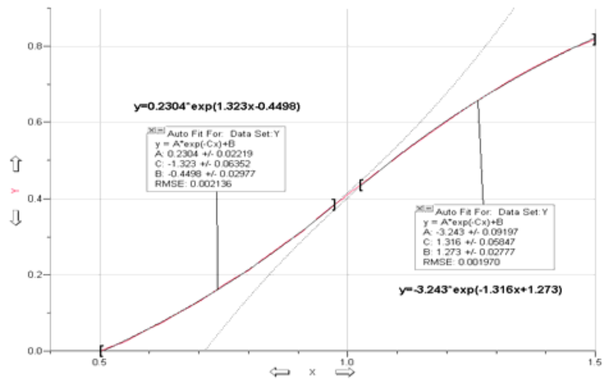

Figure 2a: Discrete energy level 1.

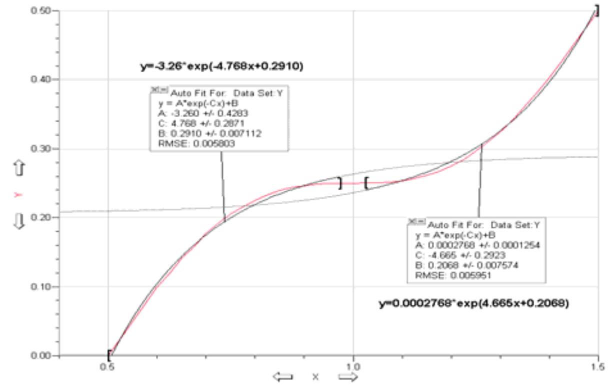

Figure 2b: Discrete energy level 2.

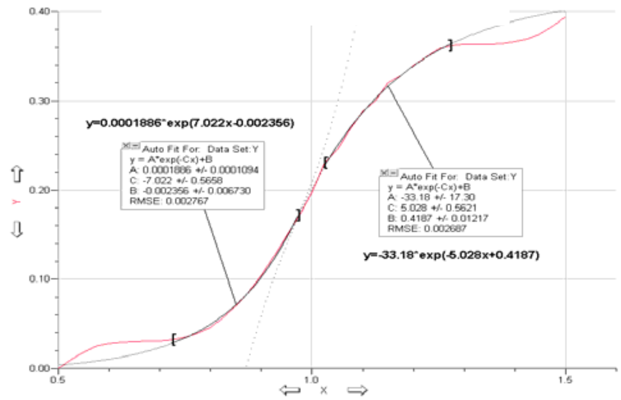

Figure 2c: Discrete energy level 3.

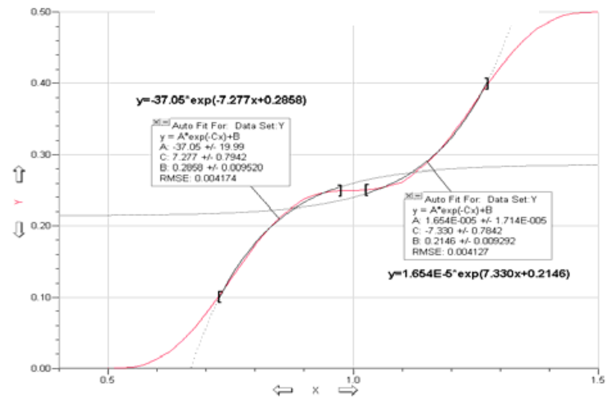

Figure 2d: Discrete energy level 4.

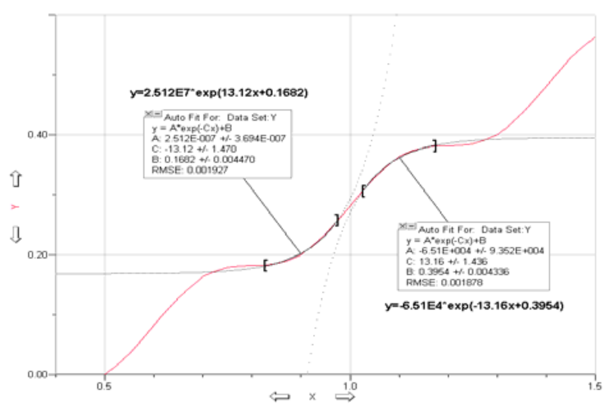

Figure 2e: Discrete energy level 5.

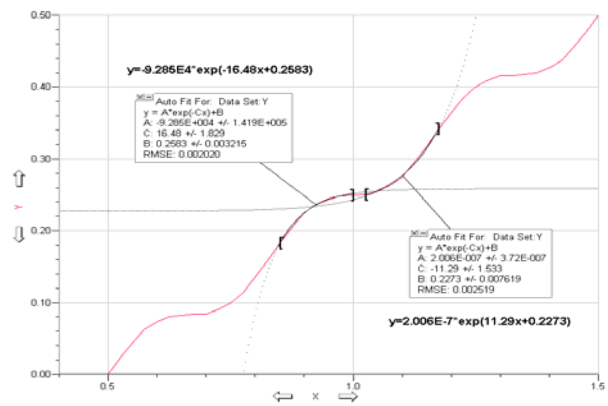

Figure 2f: Discrete energy level 6.

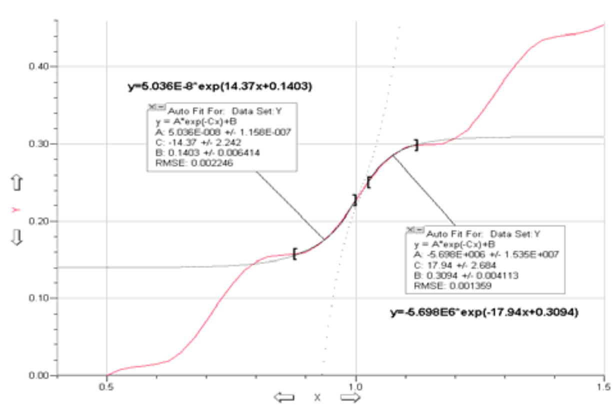

Figure 2g: Discrete energy level 7.

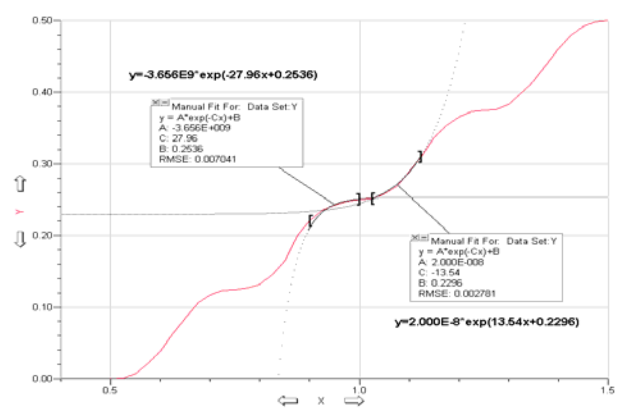

Figure 2h: Discrete energy level 8.

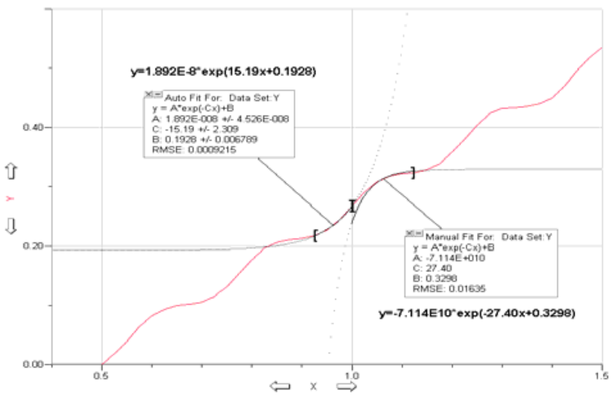

Figure 2i: Discrete energy level 9.

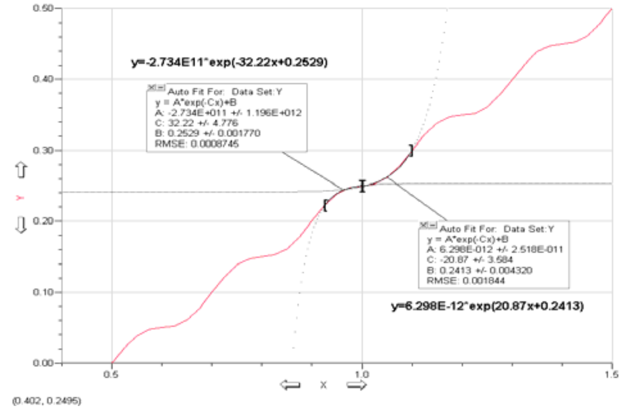

Figure 2j: Discrete energy level 10.

Figure 2:
Exponential regression equations for the first ten discrete energy levels of a one-dimensional infinite potential quantum well. Since it was later determined to only examine the center exponential regions so only those exponential regressions are shown though all were calculated, this is also for visual readability.
Respectively: (a), (b), (c), (d), (e), (f), (g), (h), (i), (j).

The output of the diodes from the second study then underwent the same treatment with exponential regression equations calculated for both the forward and reverse-biased regions of all ten diodes.

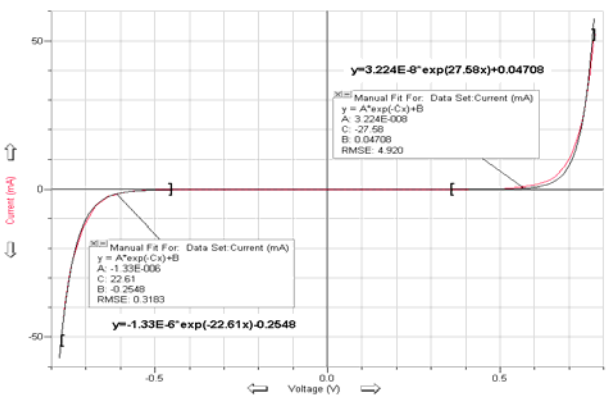

Figure 3a: Diode 1N4001.

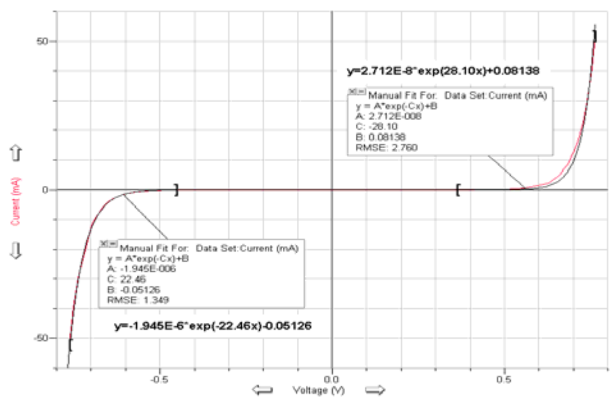

Figure 3b: Diode 1N4002.

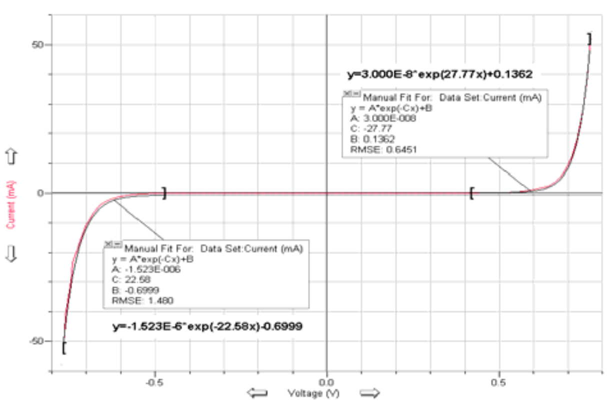

Figure 3e: Diode 1N4004.

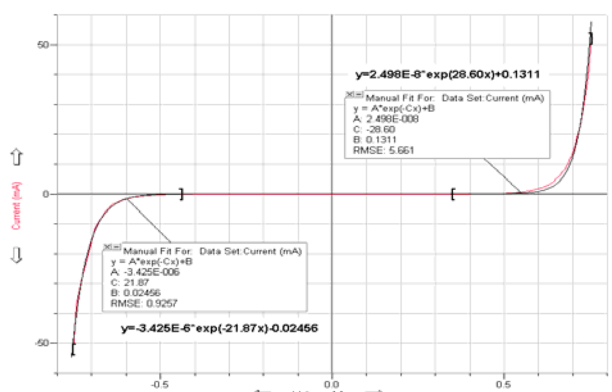

Figure 3f: Diode 1N4007.

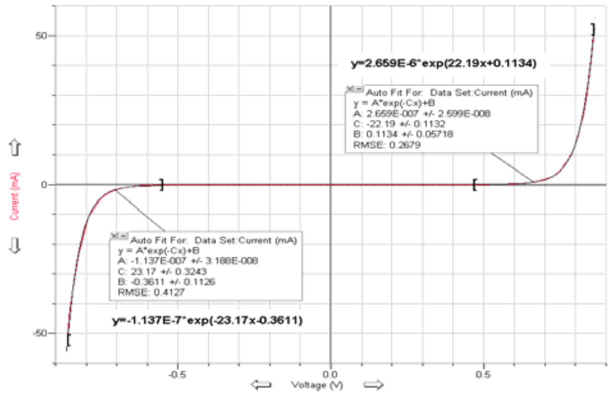

Figure 3g: Diode 1N458A..

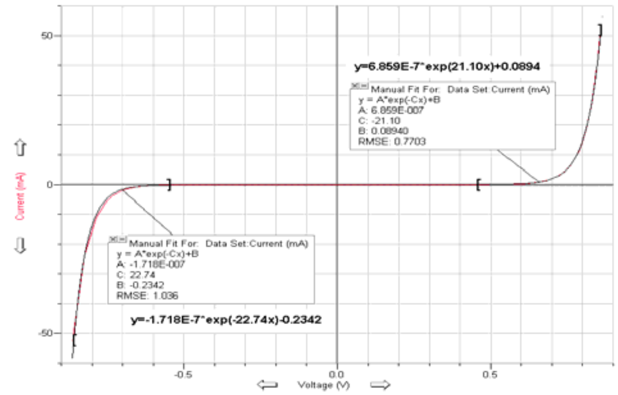

Figure 3h: Diode 1N459A.

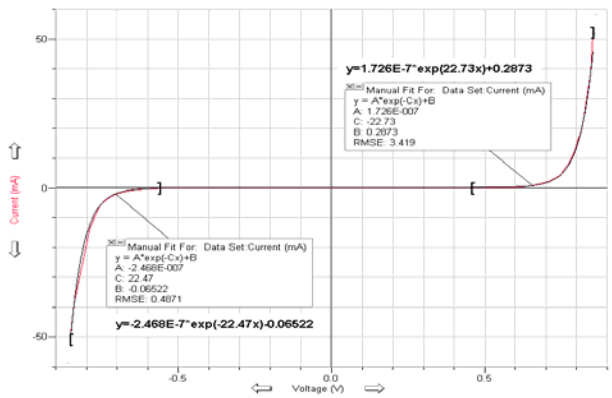

Figure 3i: Diode 1N475.

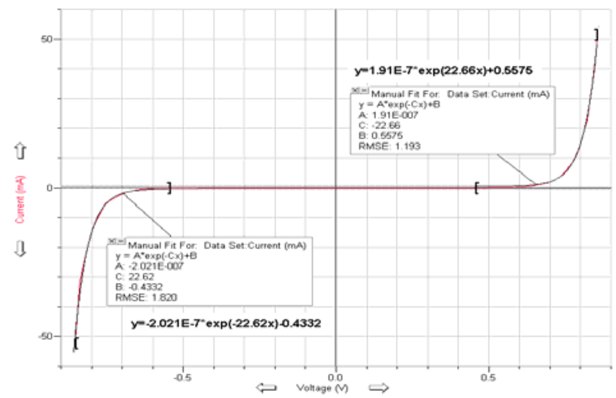

Figure 3j: Diode BAY73.

Figure 3:
Exponential regression equations for the forward and reverse-biased regions of the ten tested diode types.
Respectively: (a), (b), (c), (d), (e), (f), (g), (h), (i), (j).

Studying the exponential functions, it was apparent that conclusions could be srengthened if a correlation between the regression equations could be established. Mathematically this would involve a relationship between the general Schrödinger equation solution and the ideal diode equation as defined by the second experiment graphs.

$$\Psi(x,t) = Ae^{-i\frac{E}{\hbar}t} \quad \text{Equ. 37}$$

37: General solution to Schrödinger's eqution.

$$I = I_0(e^{\frac{V}{V_Y}} - 1) \quad \text{Equ. 38}$$

38: Ideal diode equation.

$$y = Ae^{-Cx} + B \quad \text{Equ. 39}$$

39: General exponential regression curve fit equation.

Equations 37-40:
Schrödinger's equation general solution, ideal diode equation, general exponential regression curve fit equation.

In determining which variables to compare, analytical examination centered on the equations describing the exponential behavior of the quantum well and diode systems, two in the positive direction and two in the negative direction. It was logical to begin with the ideal diode equation, Equ. 38, since its variables relate directly to data collected in experimentation. The ideal diode equation contains only the the saturation current, I_0, and the exponential factor, $e^{\frac{V}{V_T}}$. The denominator in the exponential factor, V_T, represents the thermal voltage and is defined by $V_T = \frac{kT}{q}$. The T, is indicative of the diode's temperature and since temperature was determined to be a neglected factor, only the primary coefficient, I_0, makes sense to use (Thomas, 1999). Comparing this to the Schrödinger equation, Equ. 37, therefore means using the corresponding variable which, in this case, is the coefficient before the exponential term or, A. Conclusively, this means that when comparing the regression equations of the quantum well data to the diode data, it would be most appropriate to compare the primary coefficients of the exponential regression equations, or variable A from Equ. 39 against a factor defining the data being analyze (quantum well or diode data).

Starting with the quantum well data, the center two exponential regions (one positive and one negative exponential curve) of the probability density curves were isolated for the first ten discrete energy levels to analyze in this experiment. Then, the primary coefficients from the exponential regression equations for the positive exponential region from each of the discrete energy levels was graphed (dependent variable) versus the discrete energy level from which it came (independent variable). The same was done for the negative exponential regions of the quantum well data.

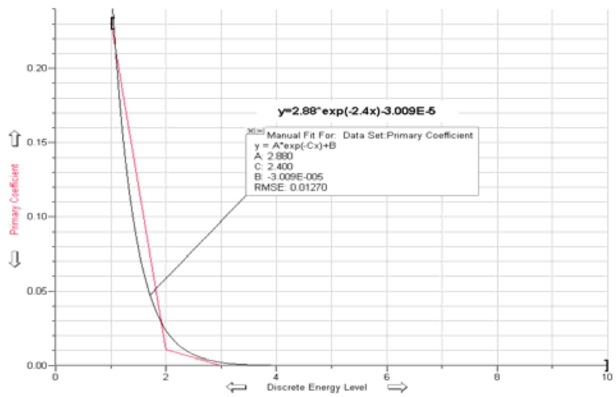

Figure 4a:
Positive exponential regression regions.

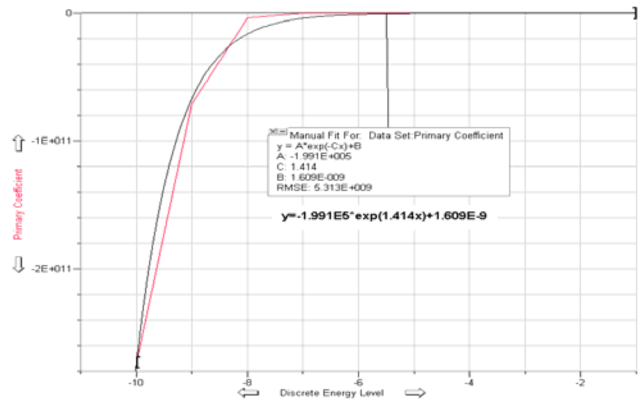

Figure 4b:
Negative exponential regression regions.

Figure 4:
Graph of primary coefficients for the positive (a) and negative (b) exponential regions of the first ten discrete energy levels.

Next, the same procedure was conducted on the exponential regression equations from the diode data. Since each diode had achieved a maximum or minimum voltage during experimentation this value was used as the independent variable against which the primary coefficients would be graphed. The positive and negative exponential region coefficients were graphed separately. The graphs for the two regions consisted of the primary coefficients of the regression equations (dependent variable) versus the maximum or minimum (based on if the graph was for the forward or reverse-bias) voltage achieved by the diode in question (independent variable).

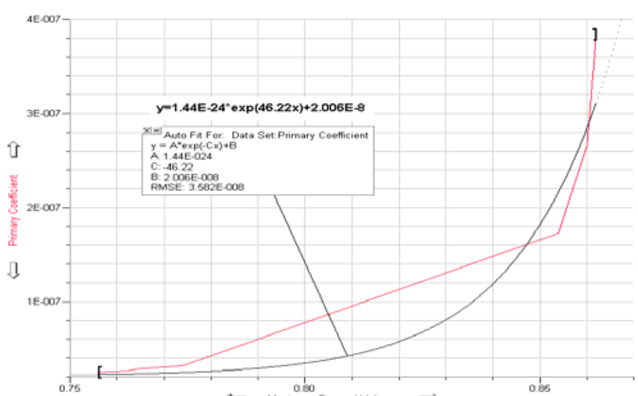

Figure 5a:
Positive exponential regression regions

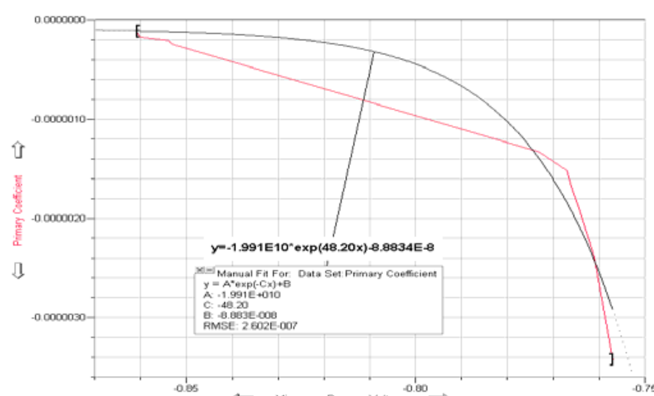

Figure 5b:
Negative exponential regression regions

Figure 5:
Graph of primary coefficients for the positive (a) and negative (b) exponential regions of the ten diodes.

The result of this analysis supported the claim of a relationship between the primary coefficients as evidenced by their markedly exponential behavior for both the positive and negative exponential regions of the quantum well and diode data. Visual synonymy established, analytical assessment called to conclude with a mathematical relationship between the discrete energy levels and the diodes. The two exponential equations describing the behavior of the newest graphs were taken and manipulated.

$$y = 2.88\times e^{-2.4n} - 3.009\times 10^{-5}$$ *Equ. 40*

41:Quantum well exponential equation from Figure 4a.

$$y = (1.44\times 10^{-24})\times e^{46.22v} + 2.006\times 10^{-8}$$ *Equ. 41*

42:Diode data exponential eauation from Fiaure 5a.

$$46.22v + 2.4n = 55.955$$ *Equ.42*

43: Quantum well-diode positive region correlation equation.

$$y = (-1.991\times 10^{5})\times e^{-1.414n} + 1.609\times 10^{-9}$$ *Equ. 43*

44: Quantum well exponential equation from Figure 4b.

$$y = (-1.991\times 10^{10})\times e^{48.20v} - 8.883\times 10^{-}$$ *Equ. 44*

45: Diode data exponential equation from Figure 5b.

$$48.20v + 1.414n = -11.512$$ *Equ. 45*

46: Quantum well-diode negative region correlation equation.

Equations 40-45:
Calculating the mathematical relationship between the positive (Equs. 40-42) and negative (Equs. 43-45) exponential regions of quantum well and diode data.

Conclusion

Based on these final calculations, the conclusion of the second aspect of this research was that the p-n junction in the form of a general purpose diode is a plausible physical structure to model the behavior of an infinite potential quantum well due to the synonymy between the exponential regions of the probability density function for different discrete energy levels of the quantum well and the exponential output of current versus voltage graphs for variable diodes. Mathematically, a relationship was determined where, using Equ. 42 or Equ. 45, a discrete energy level can be studied by calculating the minimum or maximum voltage (depending if the forward or reverse-biased direction is being studied) needed in a diode to mimic the properties of an electron in the center of a quantum well at that discrete energy level.

Quantum nanostructures are complex materials which currently can only be studied at institutions where advanced technology can be utilized. However, through this research, it has been shown that a simple electrical component, the diode, can exhibit some of the properties of a quantum well. Therefore, it is plausible that experiments, primarily mathematical or theoretical in nature, could be conducted using the properties of a p-n junction before being applied to the quantum structure itself.

Since this research produced a general mathematical equation, the immediate steps to be taken for future research involve validation of this equation by repeating the experiment to ensure the same results are collected. Also, using the equations themselves to calculate the types of diodes needed to mimic the behavior of quantum wells and ensuring that the diodes do exhibit the necessary characteristics would be a vital step towards corroborating these results. Next, it is necessary to explore other properties of diodes which may be indicative of the behavior of electrons in quantum wells with the continued goal of being able to utilize these electrons in quantum computing techniques. This study centers around an electron residing in the theoretical structure of a one-dimensional infinite potential quantum well. With this basis established, the research now aims to expand the scope of this research for a realistic three-dimensional well, incorporating the ability of an electron to propogate inside the well and eventually to leave it entirely (this study normalized the probability of the electron being somwhere within the well to 1, assuring that the electron would stay in the well). The hydrogen atom, for example, is an attractive structure to study due to the fact that it contains a single electron. Some hypotheses state that the key to understanding the movement of an electron lies within studying its behavior within the hydrogen atom (Fong, 1962).

Some currently notable research in this field regard optimizing the p-n junction itself. North Carolina State University is working with making p-n junctions with "in-situ phosphorus doped silicon germanium alloys." The scientists involved hope to find that the custom made junctions exhibit amplified electrical properties such as "active carrier transport" (Kang, 2004). At the Florida Institute of Technology, higher level mathematical techniques are being used to chart the nonlinear characteristics of the p-n junction diode when exposed to radiation beams (Shi, 2004). The University of Cambridge has an entire semiconductor physics group whose goals include "fundamental quantum transport phenomena using advanced semiconductor structures." Some of their individual projects include one, two, and three-dimensional electron transport as well as quantum structure formation by regrowth and quantum computation through a variety of techniques (Pepper et al., 2006).

Apart from immediate innovations, these collective experiments seek a higher goal. The relationship between classical physics and chaos and between classical physics and quantum physics has been established, thought not necessarily understood. The third link to the triangle, between chaos and quantum physics, has yet to be established. To this end, specific analysis of the nonlinear regions of quantum structures or of structures with analagous properties (ie. the diode) would involve such techniques as bifurcation graphing and power spectra analysis (Cross, 2005). Current research shows that these two methods could be used to model future progression into chaos of a system given initial conditions. It is already known that over reverse-biasing a diode in a circuit can lead to chaotic behavior. Through these precedents and building upon the current research presented here, the ultimate goal is to build up a basis to define the third link, quantum chaos.

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